End-to-End Performance of Randomized Distributed Space-Time Codes

Trung Q. Duong[‡†], Özgü Alay[†], Elza Erkip[†], and Hans-Jürgen Zepernick[‡]
[‡]Blekinge Institute of Technology, Sweden
Email: {dqt,hjz}@bth.se
[†]Polytechnic Institute of New York University, U.S.A.
E-mail: {dtrung,elza}@poly.edu, ozgu@vision.poly.edu

***Abstract*—The exact expressions for symbol error probability and outage probability of randomized distributed space-time codes (RDSTC) under Rayleigh fading are derived. The diversity gain derived in the literature uses the Chernoff bound which may not be achievable in general. Utilizing the exact expression, the achievable diversity gain of RDSTCs is validated. The analytical expressions are verified by Monte-Carlo simulations.**

I. Introduction

Distributed space-time codes (DSTCs) have remarkably gained interest in the research community due to the possibility of achieving spatial diversity gain through the use of relays [1]–[4]. Such code designs require central control to coordinate all the relays in the network. However, in a large-scale distributed network enabling such a central control would require a prohibitive amount of overhead.

To overcome the above challenge, randomized distributed space-time codes (RDSTCs) have been proposed in [5], [6]. The key idea behind RDSTC is to alleviate the need for centralized control. In this context, in [5] m-Group RDSTC has been proposed where the decoded set is divided into m groups. Each relay inside a group is independently and randomly preassigned a space-time code with random phase, and space-time codes are re-used among the groups. A more general scheme is proposed in [6] where each active relay randomly and independently combines the linear columns of a space-time code matrix. Specifically, after correctly decoding the source symbols, a relay constructs a predefined space-time code matrix and then independently multiplies it with a random vector $\boldsymbol{r}$. The RDSTC is specified by the random matrix $\boldsymbol{\mathcal{R}}$ whose size is $L \times N$ where L is the number of antennas of the underlying space-time code and N is the number of active relays. Depending on the statistical distribution of $\boldsymbol{\mathcal{R}}$ chosen, e.g., complex Gaussian, real Gaussian, uniform phase, random antenna selection, different kinds of RDSTCs can be constructed. The selection of randomization matrix R has been discussed in details in [6]. It has been shown that the m-Group RDSTC in [5] is a special type of [6] corresponding to the case of the random matrix $\boldsymbol{\mathcal{R}}$ taking the form of a block-diagonal matrix.

[0]This work is supported in part by National Science Foundation (NSF) under awards 0520054, 0430885, 0905446 and the New York State Center for Advanced Technology in Telecommunications (CATT). The work is also supported by Wireless Internet Center for Advanced Technology (WICAT).

The performance of RDSTCs has been investigated in [7]–[9]. In particular, the outage probability (OP) of RDSTC when $\boldsymbol{\mathcal{R}}$ has uniform-phase distribution has been presented in [7], [8]. In [9], assuming $\boldsymbol{\mathcal{R}}$ follows random antenna selection, the approximated OP for RDSTC has been developed. It is worth to mention references [10], [11], although these RDSTCs are different from those proposed in [6]. To be specific, in [10], [11] an active relay does not generate the underlying space-time code but multiplies the received vector of symbols with the random spreading matrix. With this strategy, the decoding complexity at the destination is increased compared to [6]. The performance of this RDSTC for both decode-and-forward (DF) and amplify-and-forward (AF) relays has also been derived for the approximation of OP.

To the best of our knowledge there is no published work concerning the exact performance of RDSTC with complex Gaussian distribution of random matrix $\boldsymbol{\mathcal{R}}$. More recently, this type of RDSTC has been intensively applied for video multicast [12], [13] due to its low encoding and decoding complexity and high performance. Motivated by all of the above, in this paper, we analyze the end-to-end (e2e) performance of RDSTC with complex Gaussian distribution for $\boldsymbol{\mathcal{R}}$. Specifically, we derive exact expressions of the symbol error probability (SEP) and OP over Rayleigh fading channels. Moreover, unlike [6] where the diversity gain is bounded using the Chernoff bound which is not generally tight, we derive the diversity gain from the exact expression of SEP. The derived diversity matches that of [6] suggesting the tightness of their results. Some interesting observations of the asymptotic performance are also obtained for large-scale networks. Finally, we provide the numerical results to verify our analysis.

The paper is organized as follows. The system model for a RDSTC with complex Gaussian distribution is described in Section II. In Section III, the anlytical expressions for SEP and OP are derived. Numerical results are given in Section IV. Finally, Section V provides concluding remarks.

Notation: A vector and a matrix are written as bold lower case and upper case letters, respectively. $\boldsymbol{I}_n$ represents the $n \times n$ identity matrix, $\mathbf{0}$ denotes the all zero matrix, and $\|\boldsymbol{A}\|_{\mathrm{F}}$ defines Frobenius norm of the matrix $\boldsymbol{A}$. $\mathbb{E}\{.\}$ is the expectation operator. We denote $\mathcal{CN}\left(\mu, \sigma^2\right)$ is the complex circularly symmetric Gaussian distribution with mean μ and variance σ^2. Let $\boldsymbol{X} \in \mathbb{C}^{m\times n}$ be the matrix-variate complex Gaussian distribution defined as $\boldsymbol{X} \sim \tilde{\mathcal{N}}_{m,n}\left(\boldsymbol{M}, \boldsymbol{\Sigma}\otimes\boldsymbol{\Phi}\right)$ if $\mathrm{vec}(\boldsymbol{X}^\dagger)$ is mn-variate complex Gaussian distributed with mean $\mathrm{vec}(\boldsymbol{M}^\dagger)$ and covariance $\boldsymbol{\Sigma}^T\otimes\boldsymbol{\Phi}$. The PDF of $\boldsymbol{X}$ is given by

$$p_{\boldsymbol{X}}\left(\boldsymbol{X}\right) = \pi^{-mn}\det\left(\boldsymbol{\Sigma}\right)^{-n}\det\left(\boldsymbol{\Phi}\right)^{-m} \times \exp\left[-\operatorname{tr}\{\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{X}-\boldsymbol{M}\right)\boldsymbol{\Phi}^{-1}\left(\boldsymbol{X}-\boldsymbol{M}\right)^\dagger\}\right] \quad (1)$$

II. System and Channel Models

In this section, we briefly introduce the system and channel model of RDSTC with respect to the distribution of $\boldsymbol{\mathcal{R}}$ being complex Gaussian. Consider a wireless network with $K+2$ single-antenna nodes consisting of a source, a destination, and a set of K DF relays, $\mathsf{C} = \{\mathsf{C}_1, \mathsf{C}_2, \ldots, \mathsf{C}_K\}$. All terminals operate in half-duplex mode, i.e., they cannot transmit and receive simultaneously. In addition, we assume perfect synchronization and quasi-static fading, i.e., the channel remains unchanged for a block and varies independently every block. The node synchronization of RDSTC is discussed in details in [14].

In the first-hop transmission, the source broadcasts the information to all of the K relays in the network with average transmit power per symbol P_s. The nodes that can correctly decode the source's message form the relaying set. Due to the error-prone fading channel between the source and each relay, mobility, or node failure, the relaying set will be random. Hence, RDSTC is the perfect choice for such scenario.

Let us denote a $P\times 1$ vector $\boldsymbol{s}$ as the source's message transmitted in the first hop. After correctly decoding $\boldsymbol{s}$, each relay will construct a space-time code $\boldsymbol{\mathcal{G}}$ [15] which has been predefined for the entire network. Here, $\boldsymbol{\mathcal{G}}$ is a $P\times L$ space-time code matrix and L is the number of antennas of the underlying space-time code. Then, the i-th relay will transmit a vector $\boldsymbol{x}_i$ consisting of a block of P symbols which is the random linear combinations of columns of $\boldsymbol{\mathcal{G}}$ to the destination. Let $\boldsymbol{r}_i$ be the $L\times 1$ vector whose elements are the complex Gaussian random variables, we have

$$\boldsymbol{x}_i = \boldsymbol{\mathcal{G}}\boldsymbol{r}_i \quad (2)$$

Assuming that there are m relays active in the second hop, the received signal at the destination can be shown as

$$\boldsymbol{y} = \boldsymbol{\mathcal{G}}\boldsymbol{\mathcal{R}}\boldsymbol{h}_2 + \boldsymbol{n} \quad (3)$$

where $\boldsymbol{\mathcal{R}} = [\boldsymbol{r}_1\boldsymbol{r}_2\ldots\boldsymbol{r}_m]$, $\boldsymbol{h}_2$ is an $m\times 1$ vector which is composed of the channel gains from all m relays to the destination, and $\boldsymbol{n}$ is the additive white Gaussian noise (AWGN) vector whose elements follow $\mathcal{CN}\left(0, N_0\right)$. At the receiver, the equivalent channel gain (which includes the channel gain and the randomization matrix, $\boldsymbol{\mathcal{R}}\boldsymbol{h}_2$) is estimated using pilot signals [6]. In other words, the receiver does not need to know $\boldsymbol{\mathcal{R}}$ but rather estimates $\boldsymbol{\mathcal{R}}\boldsymbol{h}_2$. Therefore, decoders already designed for space-time code reception can be directly used for R-DSTC decoding.

It is important to note that since each active relay generates a random vector $\boldsymbol{r}_i$ independently from each other, a $L\times m$ matrix $\boldsymbol{\mathcal{R}}$ can be referred to as the randomization matrix. Depending upon the distribution of $\boldsymbol{\mathcal{R}}$, different classes of RDSTC can be obtained as in [6]. In this paper, we focus on the case of complex Gaussian distribution. Without loss of generality, we assume that $\boldsymbol{\mathcal{R}} \sim \tilde{\mathcal{N}}_{L,m}\left(\boldsymbol{0}, \boldsymbol{I}_L\otimes\boldsymbol{I}_m\right)$.

III. End-to-End Performance Analysis

Let us denote $\mathcal{D}_m$ as a decoding set of relays. This $\mathcal{D}_m$ set consists of m relays among the set of K relays, $\mathsf{C} = \{\mathsf{C}_1, \mathsf{C}_2, \ldots, \mathsf{C}_K\}$, which are active in the second hop and considered to be the subset of C. The e2e performance of RDSTC can be given by

$$P_{\text{e2e}}\left(\mathcal{A}\right) = \sum_{m=0}^{K}\sum_{\mathcal{D}_m}\Pr\left(\mathcal{A}|\mathcal{D}_m\right)\Pr\left(\mathcal{D}_m\right) \quad (4)$$

where $\mathcal{A}$ can be either an error or outage event. If $\mathcal{A}$ is an error event then we define $\Pr\left(\mathcal{A}|\mathcal{D}_m\right)$ as the SEP P_{e}. If $\mathcal{A}$ is an outage event then we denote $\Pr\left(\mathcal{A}|\mathcal{D}_m\right)$ as the OP P_{out}. Here, $\Pr\left(\mathcal{D}_m\right)$ denotes the probability that the subset $\mathcal{D}_m$ is the decoding set which is clarified in the sequel.

The i-th relay receives source information and hence is active for transmission in the second hop if $\frac{1}{2}\log_2\left(1+|h_{1i}|^2\gamma_0\right) \geq R_1$ where h_{1i} is the channel coefficient from the source to the i-th relay and R_1 is the transmission rate of the source. Then, we have

$$\begin{aligned}\Pr\left(\mathcal{D}_m\right) &= \prod_{i\in\mathcal{D}_m}\Pr\left(|h_{1i}|^2 \geq \varphi_1\right)\prod_{j\notin\mathcal{D}_m}\Pr\left(|h_{1j}|^2 < \varphi_1\right) \\ &= \prod_{i\in\mathcal{D}_m} e^{-\varphi_1/\Omega_{1i}}\prod_{j\notin\mathcal{D}_m}\left(1-e^{-\varphi_1/\Omega_{1j}}\right)\end{aligned} \quad (5)$$

where $\varphi_1 = \frac{2^{2R_1}-1}{\gamma_0}$ and Ω_{1i} is the channel mean power of the link from the source to the i-th relay.

A. Moment Generating Function

In order to analyze the SEP and the OP of RDSTC, we need to compute the moment generating function (MGF) of the instantaneous received signal-to-noise ratio (SNR) at the destination. As can be observed from (3), the RDSTC scheme can be viewed as deterministic space-time code $\boldsymbol{\mathcal{G}}$ transmitted over the randomized channel $\boldsymbol{\mathcal{R}}\boldsymbol{h}_2$. If the underlying space-time code is constructed in an orthogonal design, the maximum likelihood (ML) decoding becomes the symbolwise decoding, i.e., each transmitted symbol s_k, $k = 1, 2, \ldots P$, can be independently decomposed from each other [15]. Then, the instantaneous received SNR at the destination is given by [6]

$$\gamma = \gamma_0\left\|\boldsymbol{\mathcal{R}}\boldsymbol{h}_2\right\|_{\text{F}}^2 \quad (6)$$

where $\gamma_0 = P_s/N_0$ is the average SNR. It has been assumed that each element of $\boldsymbol{\mathcal{R}}$ is a complex Gaussian random variable with zero mean and unit variance, i.e, $\mathcal{CN}\left(0,1\right)$ [6]. Hence, the PDF of $\boldsymbol{\mathcal{R}}$ can be expressed as follows:

$$p_{\boldsymbol{\mathcal{R}}}\left(\boldsymbol{\mathcal{R}}\right) = \pi^{-Lm}\operatorname{etr}\{-\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^\dagger\} \quad (7)$$

where $\mathrm{etr}\{\cdot\}$ is the exponential trace. From (6) and (7), the MGF of γ, $\phi_\gamma(s) = \mathbb{E}_\gamma\{\exp(-\gamma s)\}$ can be computed as

$$\phi_\gamma(s) = \mathbb{E}_{\boldsymbol{h}_2}\left\{\pi^{-Lm}\int_{\boldsymbol{\mathcal{R}}}\mathrm{etr}\Big[-\boldsymbol{\mathcal{R}}\left(\boldsymbol{I}_m + s\gamma_0\boldsymbol{h}_2\boldsymbol{h}_2^\dagger\right)\boldsymbol{\mathcal{R}}^\dagger\Big]\right\} \quad (8)$$

The inner quantity of the expectation operation given in (8) can be easily evaluated at once with the help of (1), yielding

$$\phi_\gamma(s) = \mathbb{E}_{\boldsymbol{h}_2}\left\{\det\left(\boldsymbol{I}_m + s\gamma_0\boldsymbol{h}_2\boldsymbol{h}_2^\dagger\right)^{-L}\right\} \quad (9)$$

where (9) follows from the fact that $\det(\boldsymbol{A}^{-1}) = \det(\boldsymbol{A})^{-1}$. Finally, by utilizing the Sylvester's determinant theorem, i.e., $\det(\boldsymbol{I}+\boldsymbol{AB}) = \det(\boldsymbol{I}+\boldsymbol{BA})$, we have

$$\phi_\gamma(s) = \mathbb{E}_{\|\boldsymbol{h}_2\|_{\mathrm{F}}^2}\left\{\left(1 + s\gamma_0\|\boldsymbol{h}_2\|_{\mathrm{F}}^2\right)^{-L}\right\} \quad (10)$$

We next consider the MGF in two cases with independently and identically distributed (i.i.d.) and independently but not identically distributed (i.n.i.d.) fading channels.

1) MGF with i.i.d. Rayleigh fading channels: In this case, without loss of generality, we assume that the fading coefficient from each relay to the destination is a complex Gaussian random variable with zero mean and unit variance. Hence, $\|\boldsymbol{h}_2\|_{\mathrm{F}}^2$ is the chi-square distribution with $2m$ degrees of freedom and mean m, namely

$$p_{\|\boldsymbol{h}_2\|_{\mathrm{F}}^2}(x) = \frac{x^m}{\Gamma(m)}\exp(-x) \quad (11)$$

From (10) and (11), the MGF of γ can be rewritten as follows:

$$\begin{aligned}\phi_\gamma(s) &= \int_0^\infty \frac{1}{\Gamma(m)}\frac{x^m}{(1+s\gamma_0 x)^L}\exp(-x)\,dx \\ &= \left(\frac{1}{s\gamma_0}\right)^m \Psi\left(m, m-L+1; \frac{1}{s\gamma_0}\right) \quad (12)\end{aligned}$$

where $\Psi(a,b;z)$ is the confluent hypergeometric function [16, eq. (9.211.4)]. Due to the fact that $z^a\Psi(a, a-b+1; z) = {}_2F_0\left(a,b;;-z^{-1}\right)$ [17, eq. 6.6.(1)], the MGF of γ given in (12) can be simplified as follows:

$$\phi_\gamma(s) = {}_2F_0(m, L;;-s\gamma_0) \quad (13)$$

2) MGF with i.n.i.d. Rayleigh fading channels: In this case, we assume that the link from each relay to the destination undergoes different channel mean power. Specifically, $\boldsymbol{h}_2$ is the $m\times 1$ complex Gaussian random vector where the n-th element follows $\mathcal{CN}(0,\Omega_{2n})$. Under this condition, the PDF of $\|\boldsymbol{h}_2\|_{\mathrm{F}}^2$ can be given as

$$p_{\|\boldsymbol{h}_2\|_{\mathrm{F}}^2}(x) = \sum_{n=1}^m \frac{\alpha_n}{\Omega_{2n}}\exp(-x) \quad (14)$$

where α_n is the coefficient defined as

$$\alpha_n = \prod_{\substack{k=1\\k\neq n}}^m\left(\frac{\Omega_{2n}}{\Omega_{2n}-\Omega_{2k}}\right)$$

By substituting (14) to (10) and after some manipulations, we obtain $\phi_\gamma(s)$ for the case of i.n.i.d. Rayleigh fading channels as

$$\phi_\gamma(s) = \sum_{n=1}^m \frac{\alpha_n}{s\gamma_0\Omega_{2n}}\exp\left(\frac{1}{s\gamma_0\Omega_{2n}}\right)E_L\left(\frac{1}{s\gamma_0\Omega_{2n}}\right) \quad (15)$$

where $E_n(z)$ is the generalized exponential integral function defined as $E_n(z) = \int_1^\infty \frac{e^{-zt}}{t^n}dt$.

B. Symbol Error Probability

Using the MGF approach, the average SEP of RDSTC for M-PSK modulations can be expressed as [18]

$$P_{\mathrm{e}} = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\phi_\gamma\left(\frac{g}{\sin^2\theta}\right)d\theta \quad (16)$$

where g is the modulation constant denoted as $g = \sin\left(\frac{\pi}{M}\right)^2$. Substituting (13) and (15) in (16) and combining with (5) and (4), the e2e SEP of RDSTC over i.i.d. and i.n.i.d. Rayleigh fading channels can be given by, respectively

$$\begin{aligned}P_{\mathrm{e2e}}^{\mathrm{iid}}(\mathsf{SEP}) &= \sum_{m=0}^K\sum_{\mathcal{D}_m}\frac{1}{\pi}\prod_{i\in\mathcal{D}_m}e^{-\varphi_1/\Omega_{1i}}\prod_{j\notin\mathcal{D}_m}\left(1-e^{-\varphi_1/\Omega_{1j}}\right) \\ &\times\int_0^{\pi-\frac{\pi}{M}}{}_2F_0\left(m, L;;-\frac{g}{\sin^2\theta}\gamma_0\right)d\theta \quad (17)\end{aligned}$$

$$P_{\mathrm{e2e}}^{\mathrm{inid}}(\mathsf{SEP}) = \sum_{m=0}^K\sum_{\mathcal{D}_m}\frac{1}{\pi}\prod_{i\in\mathcal{D}_m}e^{-\varphi_1/\Omega_{1i}}\prod_{j\notin\mathcal{D}_m}\left(1-e^{-\varphi_1/\Omega_{1j}}\right)$$

$$\times\sum_{n=1}^m\int_0^{\pi-\frac{\pi}{M}}\frac{\alpha_n\sin^2\theta}{g\gamma_0\Omega_{2n}}\exp\left(\frac{\sin^2\theta}{g\gamma_0\Omega_{2n}}\right)E_L\left(\frac{\sin^2\theta}{g\gamma_0\Omega_{2n}}\right)d\theta \quad (18)$$

Since the generalized hypergeometric and exponential integral functions are available in many common mathematical softwares packages such as MATHEMATICA and MAPLE, we can readily evaluate the SEP performance of RDSTC using some simple numerical integration techniques. Although we consider only M-PSK modulation in this paper, the SEP of RDSTC can be obtained for a wide variety of modulation schemes [18].

C. Achievable Diversity Order

Since the diversity gain derived in [6] utilizes a Chernoff bound, it may not be an achievable diversity gain. Next, we derive the diversity gain of RDSTC system taking into account the exact expression derived in previous subsection. For a fair comparison with the work in [6], we consider only the effect of the second-hop from the relays to the destination. Within this context, the diversity gain is given by [18]

$$d \triangleq \lim_{\gamma_0\to\infty}\frac{-\log P_{\mathrm{e}}}{\log\gamma_0} = \lim_{\gamma_0\to\infty}\frac{-\log\phi_\gamma(g)}{\log\gamma_0} \quad (19)$$

It is clear that the diversity gain does not depend on the channel mean power of link fading. Hence, to simplify the derivation, we only focus on the i.i.d. case. Combining (13) and (19), we obtain

$$d = \underbrace{\lim_{\gamma_0 \to \infty} \frac{-\log (g\gamma_0)^{-m}}{\log \gamma_0}}_{L_1} + \underbrace{\lim_{\gamma_0 \to \infty} \frac{\Psi\left(m, m-L+1; \frac{1}{g\gamma_0}\right)}{\log \gamma_0}}_{L_2} \tag{20}$$

It is easy to see that the first summand of (20) is m. To evaluate the second summand, let us apply the Maclaurin series expansion of the confluent hypergeometric functions

$$\Psi(a, b; z) = \frac{(b+az)\,\Gamma(-b)}{\Gamma(1+a-b)} + \frac{z^{1-b}\Gamma(b-1)}{\Gamma(a)} + \dots \tag{21}$$

As can clearly be seen from (21), $\Psi(a, b; z)$ converges to a finite value if $1-b \geq 0$ as z tends to be small. In other words, the second summand of (20), L_2, converges to zero value if $m \leq L$ as γ_0 goes to infinity. Hence, the achievable diversity gain of RDSTC is given by

$$d = m \quad \text{if } m \leq L \tag{22}$$

To derive the diversity gain for the case $m \geq L$, let us utilize the permutation symmetry of the generalized hypergeometric function, i.e., ${}_2F_0(m, L; ; -g\gamma_0) = {}_2F_0(L, m; ; -g\gamma_0)$ leading the diversity gain d in (20) as

$$d = \underbrace{\lim_{\gamma_0 \to \infty} \frac{-\log\left(\frac{1}{g\gamma_0}\right)^{L}}{\log \gamma_0}}_{L_3} + \underbrace{\lim_{\gamma_0 \to \infty} \frac{\Psi\left(L, L-m+1; \frac{1}{g\gamma_0}\right)}{\log \gamma_0}}_{L_4} \tag{23}$$

Following the same approach, we have $L_4 = 0$ if $L \leq m$ as $\gamma_0 \to \infty$ resulting in

$$d = L \quad \text{if } L \leq m \tag{24}$$

By combining (23) and (24), the diversity gain of RDSTC is of order $\min(m, L)$ which validates the result given in [6].

D. Outage Probability

The OP is defined as the probability that the instantaneous SNR γ falls below a given threshold φ_2, i.e.,

$$P_{\text{out}} = \Pr(\gamma \leq \varphi_2) = F_\gamma(\varphi_2) \tag{25}$$

where $F_\gamma(\cdot)$ is the cumulative distribution function (CDF) of γ. The CDF can be obtained from the inverse Laplace transform of the MGF, i.e., $F_\gamma(\cdot) = \mathcal{L}^{-1}\{\phi_\gamma(s)/s\}$. By utilizing the MGF function of γ given in Section III-A, we can obtain $F_\gamma(\cdot)$ using simple numerical techniques. Specifically, we obtain the closed-form expression for the e2e OP performance as [19]

$$P_{\text{e2e}}(\mathsf{OP}) = \sum_{m=0}^{K} \sum_{\mathcal{D}_m} \prod_{i \in \mathcal{D}_m} e^{-\varphi_1/\Omega_{1i}} \prod_{j \notin \mathcal{D}_m} \left(1 - e^{-\varphi_1/\Omega_{1j}}\right) \frac{e^{\frac{A}{2}}}{2^Q \varphi_2} \sum_{q=0}^{Q} \binom{Q}{q} \sum_{p=0}^{P+q} \frac{(-1)^p}{\beta_p} \Re\left\{ \frac{\phi_\gamma\left(\frac{A+2\pi jp}{2\varphi_2}\right)}{\frac{A+2\pi jp}{2\varphi_2}} \right\} + O(\xi) \tag{26}$$

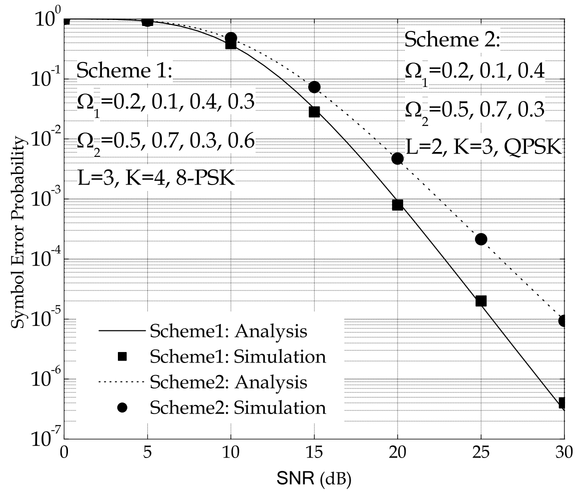

Fig. 1. SEP of RDSTC with complex Gaussian distribution.

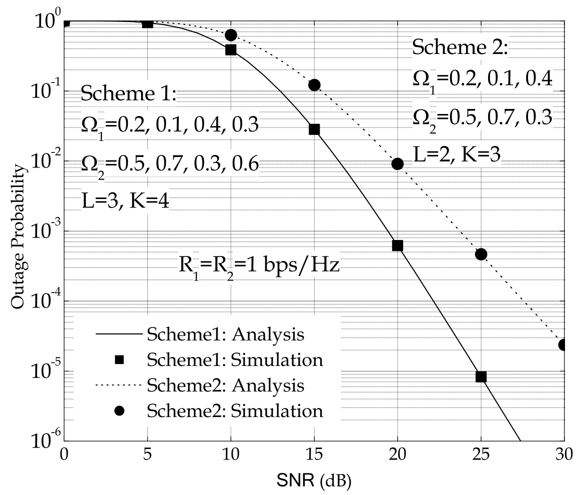

Fig. 2. OP of RDSTC with complex Gaussian distribution.

where $\beta_p =$ is a constant defined as $\beta_p = 2$ if $p = 0$ and $\beta_p = 1$ if $p = 1, 2, \dots, P$, $\Re$ is real part, and $j^2 = -1$. Here, A, P, and Q are predefined integers.

E. Asymptotic Analysis

In this section, we consider the asymptotic behavior of the performance of RDSTC as m and L tend to infinity. Again, we only focus on the i.i.d. case since the asymptotic behavior does not depend on the channel mean powers. From the MGF of γ given in (13) and using the series representation of the generalized hypergeometric functions [16, eq. (9.14.1)], [17, eq. 4.1.(1)], we have

$$\lim_{m,L\to\infty} \phi_{\gamma/(Lm)}(g) = \lim_{m,L\to\infty} {}_2F_0(m, L; -g\gamma_0/(Lm)) = \sum_{i=0}^{\infty} \frac{(-g\gamma_0)^i}{i!} \lim_{m,L\to\infty} \frac{(L)_i (m)_i}{(Lm)^i} = e^g \tag{27}$$

where $(L)_i$ and $(m)_i$ are Pochhammer symbols. The above result implies that the PDF of γ approaches to the Dirac delta

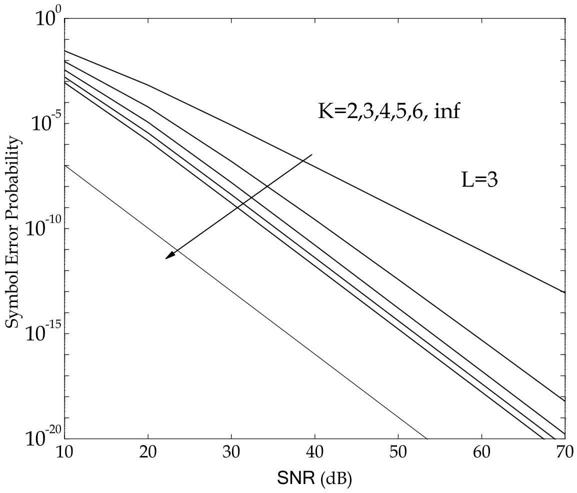

Fig. 3. SEP of RDSTC with complex Gaussian distribution.

function as L and m tends to infinity. In other words, the instantaneous SNR γ converges to Lm for large values of L and m.

IV. Numerical Results

In this section, we present the SEP and OP performance of RDSTC with complex Gaussian distribution. We also perform Monte-Carlo simulations and compare the results with analysis. For OP, we set the spectral efficiency for the first and second hops as $R_1 = R_2 = 1$ bps/Hz. For the construction of underlying space-time codes $\mathcal{G}$, we consider the general approach presented in [15] where the orthogonal space-time codes with minimum delay and maximum achievable rate were created. Specifically, for the case with $L = 2$ and $L = 3$, the space-time codes $\mathcal{G}$ are shown as follows:

$$\mathcal{G}_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \qquad \mathcal{G}_3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ -x_3^* & 0 & x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix}$$

We consider different system configurations by varying the number of antennas of underlying space-time codes L and the number of relays K as well as the channel mean power in the two following schemes:

- Scheme 1: $(L, K) = (3, 4)$, $\Omega_{11} = 0.2$, $\Omega_{12} = 0.1$, $\Omega_{13} = 0.4$, $\Omega_{14} = 0.3$, and $\Omega_{21} = 0.5$, $\Omega_{22} = 0.7$, $\Omega_{23} = 0.3$, $\Omega_{24} = 0.6$.
- Scheme 2: $(L, K) = (2, 3)$, $\Omega_{11} = 0.2$, $\Omega_{12} = 0.1$, $\Omega_{13} = 0.4$, and $\Omega_{21} = 0.5$, $\Omega_{22} = 0.7$, $\Omega_{23} = 0.3$.

Fig. 1 shows the SEP as a function of average SNR for RDSTC with complex Gaussian distribution of $\mathcal{R}$. We consider 8-PSK and QPSK modulation for Scheme 1 and 2, respectively. Fig. 2 illustrates the OP performance as a function of average SNR. As can clearly be observed from these two figures, the analytical results perfectly match with Monte-Carlo simulations. In addition, Scheme 1 outperforms Scheme 2 in both SEP and OP performance since the former achieves higher diversity gain as expected, i.e., Scheme 1: $\min(L, K) = 3$ and Scheme 2: $\min(L, K) = 2$.

For better understanding of the diversity gain of RDSTC, we plot the SEP in Fig. 3 where we fix the number of antennas of $\mathcal{G}$ as $L = 3$ and vary the number of relays $K = 2, 3, 4, 5, 6, \infty$. It is interesting to note that at the point $K = 3$ increasing the number relays results in no diversity gain since the minimum between L and K is 3. For the case $K = 2$, we can see that the diversity gain is only 2.

V. Conclusions

In this paper, we have derived the exact expression for the e2e performance of RDSTC with complex Gaussian distribution of $\mathcal{R}$. We further derive the diversity gain from the exact expression. Our result validates the previous diversity result obtained through a lower bound and shows that the diversity gain is indeed achievable. We also verify our analysis by comparing against the results of Monte-Carlo simulations.

Acknowledgment

The authors would like to thank Professor Yao Wang for her interesting discussion and constructive comments.

References

[1] J. N. Laneman and G. W. Wornell, "Distributed space–time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[2] Y. Jing and B. Hassibi, "Distributed space–time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, Dec. 2006.

[3] Z. Yi and I.-M. Kim, "High data-rate single-symbol ML decodable distributed STBCs for cooperative networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2977–2985, Aug. 2007.

[4] S. Yiu, R. Schober, and L. Lampe, "Distributed space-time block coding," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1195–1206, Jul. 2006.

[5] J. Luo, R. S. Blum, L. J. Cimini, L. J. Greenstein, and A. M. Haimovich, "Link-failure probabilities for practical cooperative relay networks," in *Proc. IEEE VTC Spring*, Stockholm, Sweden, May 2005, pp. 1489–1493.

[6] B. Sirkeci-Mergen and A. Scaglione, "Randomized space–time coding for distributed cooperative communication," *IEEE Trans. Signal Process.*, vol. 55, no. 10, pp. 5003–5017, Oct. 2007.

[7] L. Zhang and L. J. Cimini, "Outage analysis and optimization for discrete randomized distributed space–time block coding," in *Proc. IEEE GLOBECOM*, Washington DC, U.S.A., Nov. 2007, pp. 4349–4353.

[8] L. Zhang, L. J. Cimini, L. Dai, , and X.-G. Xia, "Relaying strategies for cooperative networks with minimal node communications," in *Proc. IEEE MILCOM*, Washington DC, U.S.A., Oct. 2006, pp. 1–7.

[9] S. Savazzi and U. Spagnolini, "Distributed orthogonal space–time coding: Design and outage analysis for randomized cooperation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4546–4557, Dec. 2007.

[10] D. Gregoratti and X. Mestre, "Diversity analysis of a randomized distributed space–time coding in an amplify and forward relay channel," in *Proc. IEEE ICC*, Dresden, Germany, Jun. 2009, pp. 1–5.

[11] ——, "Decode and forward relays: Full diversity with randomized distributed space–time coding," in *Proc. IEEE ISIT*, Seoul, Korea, Jun. 2009, pp. 503–507.

[12] O. Alay, P. Liu, Z. Guo, L. Wang, Y. Wang, E. Erkip, and S. Panwar, "Cooperative layered video multicast using randomized distributed space–time codes," in *Proc. IEEE INFOCOM MOVID Workshop*, Rio De Janeiro, Brazil, Apr. 2009, pp. 1–6.

[13] O. Alay, P. Liu, Y. Wang, E. Erkip, and S. Panwar, "Error resilient video multicast using randomized distributed space–time codes," in *Proc. IEEE ICASSP*, Dallas, Texas, Mar. 2010, to appear.

[14] M. Sharp, A. Scaglione, and B. S. Mergen, “Randomized cooperation in asynchronous dispersive links,” *IEEE Trans. Commun.*, vol. 57, no. 1, pp. 64–68, Jan. 2009.
[15] O. Tirkkonen and A. Hottinen, “Square-matrix embeddable space–time block codes for complex signal constellations,” *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 384–395, Feb. 2002.
[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. San Diego, CA: Academic, 2000.
[17] A. Erdelyi, *Higher Transcendental Functions*. New York: McGraw-Hill, 1953, vol. 1.
[18] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York: Wiley, 2000.
[19] J. Abate and W. Whitt, “Numerical inversion of Laplace transforms of probability distribution,” *ORSA J. Computing*, vol. 7, no. 1, pp. 36–43, 1995.