Layered Randomized Cooperation for Multicast

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***Abstract*—Cooperation of wireless users is known to provide substantial improvements in channel reliability and in end-to-end distortion. User cooperation is especially attractive for multicast since the relays are also part of the intended recipients. In this paper, we consider a randomized distributed cooperation scheme for multicasting a source signal. Using end-to-end distortion as a performance metric and assuming a delay constraint, we investigate the relation among the decoding SNR threshold, number of hops, coverage range and the distribution of end-to-end quality over users in the coverage range. In order to provide differentiated quality to users with different channel strengths, we further employ layered cooperation and illustrate the benefits.**

I. Introduction

In recent years, the demand for multimedia applications over wireless networks has been on rise due to the significant increase in both the bandwidth of wireless channels and the computational power of mobile devices. To provide efficient delivery among a group of users simultaneously, multicast has been used as an effective solution as it saves network resources by sharing a data stream across multiple receivers. However, high packet loss ratio and time variation of wireless channels, along with heterogeneity of the users, make video multicast over wireless networks a challenging problem.

User cooperation is an effective technique to combat path loss and fading where terminals process and forward signals overheard from the senders to their intended destinations [1]. Cooperation techniques have been extensively studied as a means to provide spatial diversity [2]. Cooperation of users can also be used reduce source distortion by providing unequal error protection. In our prior work [3]-[6], we studied the benefits of cooperation for point-to-point source transmission and observed significant reduction in end-to-end distortion. User cooperation is especially attractive for multicast, since the relays are among the intended recipients and hence, are free from the incentive and security concerns that may impact the deployment of cooperation for point-to-point communications.

Randomized distributed cooperation is an effective method for multi-stage broadcasting. In randomized distributed cooperation, a transmitter node initiates the broadcast by transmitting a packet and every node who can hear the source with sufficient signal-to-noise ratio (SNR), decodes and retransmits the same packet using randomized distributed space time coding. The first group excites a second group of nodes and the retransmissions continue until every node who hears the others with sufficient SNR, retransmits once. In [7], the authors studied the asymptotic behavior of such a system in a dense network and considered the propagation of information through the network.

In this paper, we study the multi-stage cooperation scheme of [7] for delivering lossy data such as multimedia signals (audio, image, and video). As the performance measure, we consider the distortion of the reconstructed signal at the receiver compared with the original source. Multimedia signals also typically have delay constraints, for example, media data has to be delivered and rendered before its scheduled playback time. Therefore, for multimedia multicast, our goal is to minimize the end-to-end distortion of the multicast receivers in a certain coverage range under a delay constraint. We investigate the effect of decoding SNR threshold, number of hops and the diversity level of the underlying space time code (STC) on the end-to-end distortion of the multicast users at a fixed coverage range.

In a multicast environment each receiver has different channel quality. In order to provide receivers signals at different distortion levels commensurate with their channel conditions, we consider layered cooperation where we transmit different layers sequentially. Note that in source coding, not all bits are equal in importance. Therefore, we apply unequal error protection (UEP) to different layers. We achieve this by choosing different SNR thresholds, diversity levels of the STC's and number of hops for different layers. We carry out our analysis using i.i.d. Gaussian source and make use of the well-known rate distortion function and the successive refinability [8] to determine the encoding-induced distortion at different source rates.

This paper is organized as follows. We introduce the system model in Section II. We study non-layered and layered cooperation and formulate the expected end-to-end distortion in Sections III and IV, respectively. We conclude the paper in Section V.

II. System Model

We study a network in which the node locations are randomly and uniformly distributed over a fixed coverage area. Specifically, we consider a dense network and study the continuum approach following the model [7], where the total relay power at each hop is fixed. We assume a squared distance path loss model and independent Rayleigh fading channels between nodes.

We assume that the broadcast transmission is initiated by the transmitter node by transmitting a packet. Every node who hears the source with sufficient signal-to-noise ratio

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above a threshold, τ, will be able to decode the packet and will retransmit. A training preamble in the message helps nodes to detect the presence of the packets, estimate the received power, and synchronize the relay transmissions. The relays use STC of dimension L and the retransmissions are done simultaneously, even though they may not be symbol synchronized. The first group excites a second group of nodes and they will activate the next group nodes. The subsequent groups of nodes that are activated are referred to as hops.

Since the nodes only use the locally available received SNR information to make transmission decisions, the network can operate in a distributed fashion. We assume that appropriate channel coding is used so that the information is correctly received as long as the received SNR is above the threshold.

In our analysis, we consider two different STC dimensions. In one extreme, we assume a high diversity regime where the relays transmit in orthogonal channels obtained by using a space time code of dimension, $L = \infty$ (orthogonal). In another extreme, we consider a low diversity transmission scheme using a space time code of dimension $L = 1$ (non-orthogonal).

We consider a system with a transmission rate R bits per channel use at each hop where the rate depends on the SNR threshold, τ, as

$$R = \log(1 + \tau) \quad (1)$$

We define a channel frame as a block of n channel uses and assume the fading amplitude is constant during a channel frame. We have k source samples to be transmitted in one channel frame leading to a bandwidth ratio of $b = n/k$ channel uses per sample. Typically the bandwidth ratio b is dictated by the application and the channel bandwidth. For example, a channel bandwidth of W Hz suggests that we have W channel uses per second. If the source is sampled at a rate of f_s samples per second and due to the delay constraints, needs to be sent at the sampling rate, the bandwidth ratio can be expressed as $b = W/f_s$ channel uses per sample. In this paper, we use b to characterize the delay constraint. A bandwidth ratio of b corresponds to transmission of bR bits for each source sample. For real i.i.d Gaussian sources with unit variance, the resulting distortion function becomes:

$$D = 2^{-2bR} \quad (2)$$

In the case of N hops, each hop has n/N channel uses available. Typically, delay constraints stipulate that k source samples need to be transmitted again in n channel uses, leading to a bandwidth ratio of $\frac{n}{kN} = \frac{b}{N}$ per hop. The resulting distortion then becomes

$$D(\tau, b, N) = 2^{-2bR/N} \quad (3)$$

Equation (3) indicates that the distortion depends on the decoding SNR threshold, bandwidth ratio and the number hops.

The end-to-end distortion at a particular location depends not only on the distortion induced by the source code, but also on the packet loss probability at that location. Section III formulates and studies the end-to-end distortion for single layer compression, while Section IV investigates layered compression where the source layers are transmitted sequentially.

III. Single Layer Transmission

In this section we discuss non-layered (or single layer) randomized distributed cooperation. We first revisit the probability of a node at (x, y) receiving the data at the j^{th} hop for a random network [7]. Then we formulate the expected distortion considering the number of hops, N, and the delay constraint represented in terms of the bandwidth ratio, b.

Let $P_j(x, y)$ denote the probability that the user at location (x,y) receives either the source or relay transmission correctly at j^{th} hop. For the first hop (i.e., source transmission), the probability can be expressed as

$$P_1 = Pr\{\|h_1(x, y)\|^2 \geq \tau\} \quad (4)$$

where $h_1(x, y)$ is the channel gain at location (x,y) for the first hop transmission and τ is the SNR threshold to be exceeded for the relay node to decode.

For the j^{th} hop ($j > 1$), we only consider the users who did not receive the information in the previous hops. All users who receive the information in the previous hop (i.e. $(j-1)^{th}$ hop) will retransmit the information. The probability of successful reception for the j^{th} hop can be expressed as,

$$\begin{aligned} P_j &= Pr\{\|h_j(x, y)\|^2 \geq \tau\} \\ &\quad \textstyle\prod_{i=1}^{i=j-1}[1 - Pr\{\|h_i(x, y)\|^2 \geq \tau\}] \end{aligned}$$

where $h_j(x, y)$ is the equivalent channel gain at location (x,y) given by

$$h_j(x, y) \sim N_c(0, \sigma_j^2(x, y)), \text{L=1 (non-orthogonal)} \quad (5)$$

$$\|h_j(x, y)\|^2 = \sigma_j^2(x, y), \text{L=}\infty \text{ (orthogonal)} \quad (6)$$

where

$$\sigma_j^2(x, y) = \int\int \tilde{P}_r P_{j-1}(x', y') l(x - x', y - y') dx' dy' \quad (7)$$

Note that $\sigma_j^2(x, y)$ is the sum of signal powers from all nodes who successfully received the information from the previous hop at location (x,y), $\tilde{P}_r$ is the relay power density and *l(d)* represents the path-loss model with

$$l(d) = \begin{cases} 1/d^2 & \text{if} \quad d > d_0 \\ 1/d_0^2 & \text{if} \quad d < d_0 \end{cases} \quad (8)$$

In general the path loss model $l(d) = 1/d^2$ arises from the free-space attenuation of electromagnetic waves, and it does not hold when d is very small leading to the model in (8).

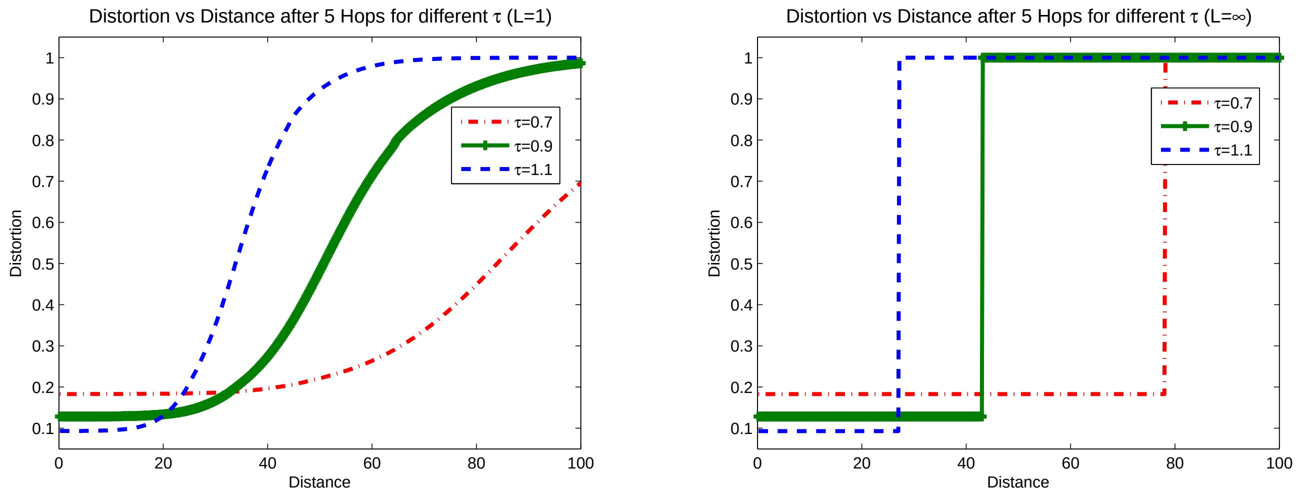
We define $P(x, y; N)$ as the probability of successful reception after N hop transmissions which can be expressed as

$$P(x, y; N) = \sum_{i=1}^{N} P_i(x, y) \quad (9)$$

Therefore, the expected distortion at location (x,y), $D_{exp}(x, y)$, is

$$D_{exp}(x, y) = P(x, y; N) D(\tau, b, N) + (1 - P(x, y; N)) \quad (10)$$

Note that since we consider a unit variance Gaussian source, when data is lost, we observe the maximum distortion, D_{max}=1.



(a) Nonorthogonal(L=1)

(b) Orthogonal (L=∞)

Fig. 1. Single Layer Cooperative Multicast: Comparison of end-to-end distortion for orthogonal and nonorthogonal relay transmission for different τ's ($P_s = 10, \tilde{P}_r = 1, d_0 = 1, N = 5, b = 8$)

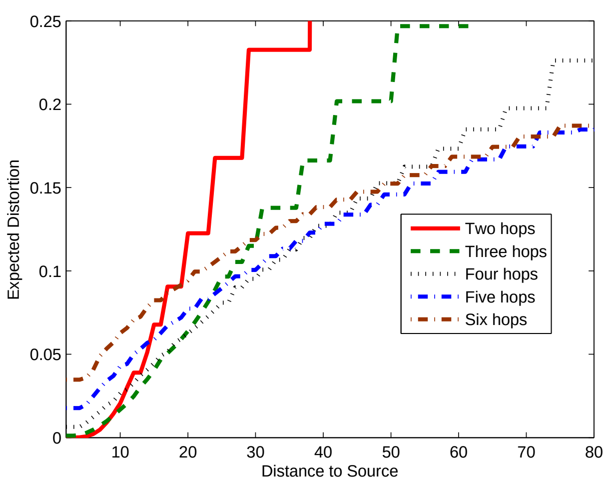


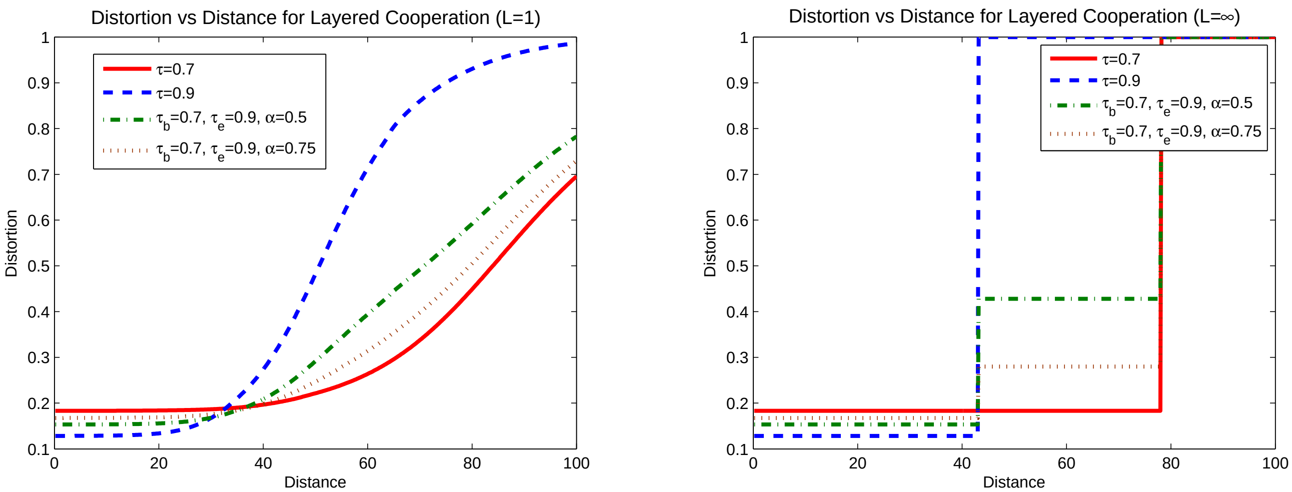
Fig. 2. Comparison of different number of hops at different coverage ranges with $P_s = 10, \tilde{P}_r = 1, d_0 = 1, b = 8$ (L=∞)

In Figure 1, for a five hop system (N=5 and b=8), we compare the end-to-end distortion as a function of distance from the source for nonorthogonal and orthogonal relay transmission and for different SNR thresholds. Note that for the orthogonal relay transmission, all nodes within a certain radius r_{th}, (r_{th}=78 for $\tau = 0.7$ in the figure) achieve the same low distortion after five hops. However, beyond this coverage distance, no node is able to receive the packets, leading to maximum distortion. On the other hand, with non-orthogonal relay transmission with the same τ, for $r < r_{th}$ the expected distortion is higher, but the source is able to reach nodes further away than r_{th} with distortion below the maximum value. Another key observation in the figure is that, as we increase τ, since we can send at a higher rate, the distortion at the closer receivers is lower. On the other hand, the corresponding coverage range defined according to some maximum tolerable distortion level also reduces.

In order to reach a larger coverage range with the same τ, we can increase the number of hops, N, thereby increasing the probability of success. However, as we increase the number of hops, the time spent for each hop reduces as well. Therefore, for a fixed coverage range, there is an optimum τ and N pair that minimizes the end-to-end distortion. Since the notion of coverage range is more clearly defined for orthogonal transmission, we explore the tradeoff between τ and N for $L = \infty$. In Figure 2, we consider orthogonal relay transmission and for each coverage range shown in the x-axis, we find the optimum τ and illustrate minimum expected distortion for different number of hops. Note that for a given coverage range, the optimum number of hops and τ are different. We observe that, while to reach short distances a small number of hops is optimum, for larger distances, we need more hops. Even with optimized number of hops, the expected distortion increases with the coverage range.

IV. Sequential Layered Cooperation

In this section, we discuss layered randomized distributed cooperation in order to provide differentiated quality for the multicast receivers based on their channel conditions. We only consider two layers, base and enhancement layer, to illustrate the main idea. We assume that we have two SNR thresholds: base layer threshold and enhancement layer threshold, τ_b and τ_e, respectively where, $\tau_e > \tau_b$. We transmit the base and the enhancement layer sequentially using TDMA (i.e. in different time slots). We assume we use α proportion of the channel frame for the base layer transmission and $1-\alpha$ proportion of the channel frame for the enhancement layer. This suggests that the base and enhancement layer bandwidth ratios are $\alpha b/N_b$ and $(1-\alpha)b/N_e$ where N_b and N_e denote the number of hops for the base and enhancement layer, respectively. We choose $N_b \geq N_e$ since we want the base layer to propagate further. We assume the fading is constant during a channel frame, hence the base and enhancement layer observe the same



(a) Nonorthogonal(L=1)

(b) Orthogonal (L=∞)

Fig. 3. Sequential Layered Cooperative Multicast: Comparison of end-to-end distortion for orthogonal and nonorthogonal relay transmissions($P_s = 10, \tilde{P}_r = 1, d_0 = 1, N_b = 5, N_e = 5, b = 8$)

fading level. At a given node, if the total received signal power is greater than τ_b we assume that we receive base layer and if the signal power is greater than τ_e we will also receive the enhancement layer.

In the previous section, we showed that the higher the SNR threshold τ, the lower the distortion is, at the expense of reduced coverage range. Hence, the main idea behind choosing different τ's and number of hops for different layers is that we want to guarantee a maximum distortion level to all users (i.e. the base layer distortion) and for users who have better channel conditions, we want to reduce the distortion even further. Also recall that as we increase the number of hops, the time spent for each hop reduces, hence the choice of (N_b, N_e) affects the distortion. Note that, for the same number of hops for the base and enhancement layers ($N_b = N_e$), by choosing different (τ_b, τ_e) we can adjust the coverage ranges and the distortion for the base and enhancement layers. For a fixed τ, the coverage area depends on how many hops we transmit. Hence, by choosing different number of hops for base and enhancement layer, we have the freedom to adjust the base and enhancement layer coverage ranges. Furthermore, for a fixed (τ_b, τ_e), by changing α, we can adjust the distortion values for the base and enhancement layer.

We next derive a general expected distortion formulation considering $(\tau_b, \tau_e, N_b, N_e, \alpha)$ and then evaluate the effect of these parameters on the performance.

We define $P_i^b(x, y)$ as the probability of successful reception of base layer at the i^{th} hop. Similarly, the probability of successful reception of enhancement layer is defined as $P_i^e(x, y)$ for the i^{th} hop. $P_i^b(x, y)$ and $P_i^e(x, y)$ can be formulated as in Section III. The probability of success after N_b hop transmission for base layer and N_e hop transmission for enhancement layer are denoted as $P_b(x, y; N_b)$ and $P_e(x, y; N_e)$, respectively. We can express these probabilities as follows:

$$P_b(x, y; N_b) = \sum_{i=1}^{N_b} P_i^b(x, y), \;\; P_e(x, y; N_e) = \sum_{i=1}^{N_e} P_i^e(x, y) \quad (11)$$

Since $\tau_b > \tau_e$, $N_b > N_e$ and the fading for the base and enhancement layers are the same, reception of the enhancement layer implies the reception of the base layer. Then, we can compute the expected distortion at location (x, y) as follows,

$$\begin{aligned} D_{exp}(x, y) \;\; = \;\; & P_e(x, y; N_e) D_{b+e}(\tau_b, \tau_e, N_b, N_e, \alpha, b) \\ & +(P_b(x, y; N_b) - P_e(x, y; N_e)) D_b(\tau_b, N_b, \alpha, b) \\ & +(1 - P_b(x, y; N_b)) \end{aligned} \quad (12)$$

where D_{b+e} is the distortion when both base and enhancement layers are received and D_b is the distortion when only base layer received. For Gaussian sources with unit variance, we can compute these distortion values as:

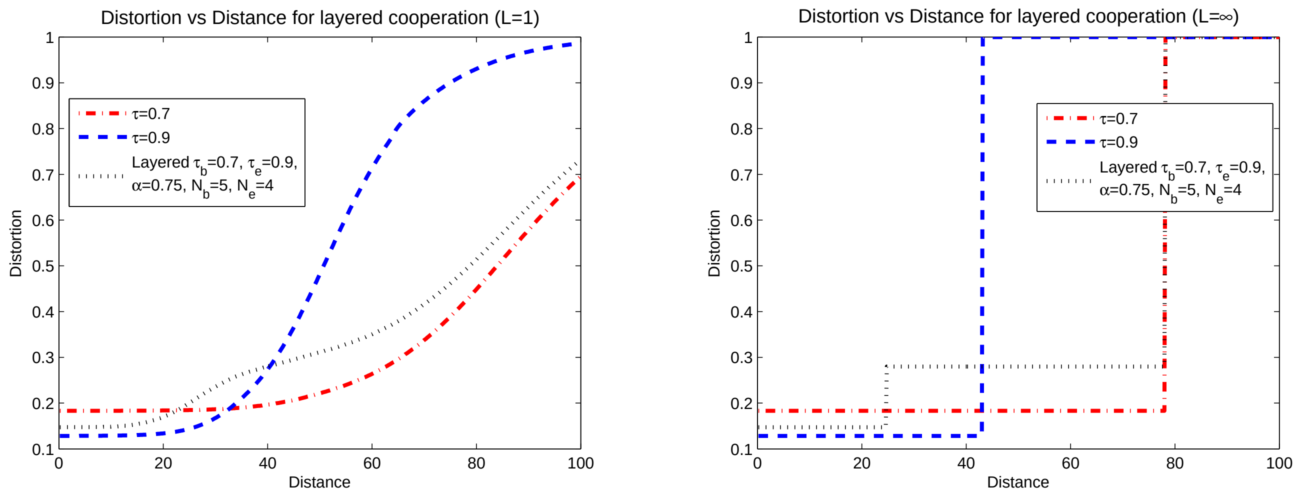
$$D_b = 2^{-2\frac{R_b \alpha b}{N_b}} \quad (13)$$

$$D_{b+e} = 2^{-2(\frac{R_b \alpha b}{N_b} + \frac{R_e(1-\alpha)b}{N_e})} \quad (14)$$

where

$$R_b = \log(1 + \tau_b) \; and \; R_e = \log(1 + \tau_e) \quad (15)$$

In Figure 3, we illustrate the effect of layered cooperation for orthogonal and nonorthogonal relay transmission for a fixed τ_b and τ_e for different α's. Here we assume both the base and enhancement layer propagate for five hops (N_b=5, N_e=5). We also plot the single layer performance for comparison. Note that, since we use equal number of hops for the base and enhancement layer, the coverage range of the base layer is similar to the single layer case with $\tau = \tau_b$ and the coverage range of the enhancement layer is similar to the single layer case with $\tau = \tau_e$. Moreover with layered cooperation, we provide lower distortion to closer nodes than



(a) Nonorthogonal(L=1)

(b) Orthogonal (L=∞)

Fig. 4. Comparison of distortion for layered cooperation with different number of hops at base and enhancement layer($P_s = 10, \tilde{P}_r = 1, d_0 = 1, N_b = 5, N_e = 4, b = 8$)

the far away nodes. Alternatively, we can think about the layered cooperation as extending the coverage range at the expense of slightly increasing the distortion for the close by nodes. Here, choosing different α, we are able to change the distortion levels at close and far away nodes.

Finally, we consider different number of hops for base and enhancement layer transmission as depicted in Figure 4. Here the single layer transmissions has $N = 5$ hops. By choosing different number of hops for base and enhancement layer, we are able to adjust the coverage range of the nodes who receive the enhancement layer. Note that compared to Figure 3 where the number of hops for base and enhancement layer are equal, the enhancement layer coverage is smaller but the distortion for the closer receivers is also lower.

V. Conclusion and Future Work

In this paper, we consider a randomized distributed cooperative multicast to reduce the end-to-end distortion in transmission of lossy sources under delay constraints. We evaluate the system for different decoding SNR thresholds and number of hops within a coverage range. We show that for a given coverage range there is an optimum τ and N which minimize the end-to-end distortion. Furthermore, in order to provide differentiated quality to users with different channel qualities, we further employ layered cooperation. We consider time division multiplex system for layering and show that by choosing the set of parameters appropriately, we have the freedom to provide different coverage ranges for the base and the enhancement layer as well as different quality levels for the users in these coverage ranges. While we assumed both base and enhancement layers use the same underlying STC, it is possible to use different STC's for different layers. As suggested in [3], for point-to-point channels, applying different diversity levels to base and enhancement layers provides another method for unequal error protection.

This paper considered sequential layered cooperation for multicast. Motivated by the benefits of superposition of layers in minimizing end-to-end distortion for point-to-point channels [3], our ongoing work considers superposition of layers for cooperative multicast.

Finally, this paper illustrated results for chosen parameters. In general, given a multicast performance metric (such as average distortion of all users), one can optimize these parameters to improve the multicast performance.

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