Cooperative Transmission of Correlated Gaussian Sources

Özgü Alay[*], Elza Erkip[*†] and Yao Wang [*]

[*]Dept. of Electrical and Computer Engineering, Polytechnic University, Brooklyn, NY

[†]Dept. of Electrical Engineering, Princeton University, Princeton, NJ

E-mails: ozgu@vision.poly.edu, {elza,yao}@poly.edu

***Abstract*— Cooperation of wireless users is known to provide substantial improvements in channel reliability and in end-to-end distortion when nodes have access to independent sources. In this paper we investigate the effect of cooperation and distributed compression when the sources are correlated. For distributed compression, realizing that each compressed stream may not reach the destination due to fading, we first derive the achievable rate-distortion pairs for Gaussian sources to have robust descriptions. Then we show how user cooperation can be coupled with robust distributed compression to further improve the end-to-end distortion for correlated sources.**

I. Introduction

Wireless networks are impaired by severe variations in signal attenuation due to fading. One effective technique to combat fading is user cooperation where terminals process and forward the overheard signal transmitted by other nodes to their intended destination [1], [2]. Cooperation techniques have been extensively studied as a means to provide spatial diversity [3]. Cooperation of users can also be used to provide reduction in source distortion by providing unequal error protection. In our prior work [4], [11], [12], [13], we studied the benefits of cooperation for independent sources and observed significant reduction in end-to-end source distortion.

In general, sources in a wireless environment can be correlated. An example for this is sensor networks. In such networks in order to reduce the communication requirements, a commonly used technique is Distributed Source Coding (DSC) for which highly correlated sources are compressed separately at each terminal and decoded jointly. From a theoretical point of view, DSC finds its foundations in Slepian-Wolf and Wyner-Ziv compression schemes [5], [6]. Distributed source coding when individual descriptions can fail to reach the destination is studied for lossless compression in [7] and for distributed estimation (CEO problem) in [8].

In this paper, we consider utilizing DSC and cooperation techniques jointly to improve the end-to-end distortion when the sources are correlated. We realize that when sources are compressed in a distributed manner, if one compressed stream does not reach the destination due to fading, the other stream is also affected. We use cooperative strategies to increase the reliability of both links, which in turn improves the robustness and efficiency of distributed compression. We use a Wyner-Ziv type setup which considers the compressed version of one of the sources as a remote side information for the compression of the other source. We make use of the results of [10] which extend the Wyner-Ziv problem to the case when the side information is lossy, and of [9] which study the rate distortion bounds when lossless side information fails to reach the destination.

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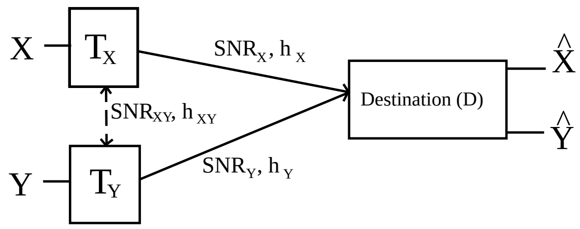

Fig. 1. Cooperative two terminal system with correlated sources

This paper is organized as follows. We introduce the system model in Section II. We study the effect of link losses on distributed compression in Section III and formulate the expected end-to-end distortion in Section IV. Section V analyzes the proposed strategies for different source and channel conditions. We conclude the paper in Section VI.

II. System model

A. Setup

Consider a cooperative system where T_x and T_y are two terminals in a wireless network communicating with a common destination. Each link has flat Rayleigh fading with instantaneous fading levels h_x, h_y and h_{xy}, and average received signal to noise ratios SNR_x, SNR_y and SNR_{xy} as shown in Figure 1. We assume a symmetric inter-user link, that is $h_{xy} = h_{yx}$. The fading levels are accurately measured at the receivers, while the transmitters are only aware of the statistics. We define a channel frame as a block of N channel uses and assume the fading is constant for multiple channel frames.

Modes	X's timeslot		Y's timeslot	
Mode A	$T_x : (bR_x, N)$		$T_y : (bR_y, N)$	
Mode B	T_x : $(bR_x, \phi_x N)$	T_y : $(1-\phi_x)N$	T_y : $(bR_y, \phi_y N)$	T_x : $(1-\phi_y)N$
Mode C	$T_x : (bR_x \vert Y, N)$		$T_y : (bR_y, N)$	
Mode D	T_x : $(bR_x \vert Y, N)$		T_y : $(bR_y, \phi_y N)$	T_x : $(1-\phi_y)N$
Mode E	T_x : $(bR_x \vert Y, \phi_x N)$	T_y : $(1-\phi_x)N$	T_y : $(bR_y, \phi_y N)$	T_x : $(1-\phi_y)N$

TABLE I

FIVE TRANSMISSION MODES, FOR EACH MODE THE COMPRESSION RATES AND THE NUMBER OF CHANNEL USES FOR EACH TERMINAL ARE SHOWN

We assume terminals T_x and T_y have access to two correlated sources X and Y respectively, which they wish to transmit to the destination with minimal expected distortion in squared error sense. The sources are zero-mean jointly Gaussian with the covariance matrix

$$K_{XY} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix} \quad (1)$$

where ρ is the correlation coefficient.

We consider time division multiple access (TDMA) among the terminals where each time slot lasts one channel frame as in [3], [11]. Each terminal may give up part of its time slot to receive help from the other. At each time slot, we send K source samples, leading to a bandwidth ratio $b = \frac{N}{K}$. For T_x, we assume the number of transmitted information bits (or source bits) per channel use is R_x. This results in a compression rate of $\overline{R}_x = \frac{NR_x}{K} = bR_x$ bits per source sample. The proportion of the channel frame used for cooperation is $(1-\phi_x)$, that is when the terminals cooperate, T_x utilizes $\phi_x N$ channel uses of its time slot, while T_y uses the remaining $(1-\phi_x)N$ to help T_x. Similar quantities can be defined for T_y as well.

B. Transmission Schemes

In order to illustrate the effects of the distributed compression and cooperation on correlated sources, we consider five transmission schemes. Below are the different modes which are summarized in Table I. The table shows the compression rate (in bits/source sample) and the number of channel uses for each terminal. In Section IV, we will optimize over these parameters to minimize the end-to-end distortion.

- $Mode\ A$: Each terminal compresses and transmits its own source directly to the destination. Hence $\phi_x = \phi_y = 1$. The terminals ignore the source correlation.
- $Mode\ B$: This mode corresponds to separate compression without utilizing the correlation, followed by cooperative transmission. For cooperation we use a decode-forward strategy as in [3], [11] where the cooperating terminal forwards the information only if it can decode it correctly (which can be checked using for example CRC). We consider sending extra parity bits rather than repetition coding at the partner [16].
- $Mode\ C$: This mode does not utilize cooperation, only distributed compression is considered. We study a specific scheme where Y is compressed independently according to the rate distortion bound and X is compressed based on Y, but in a robust fashion realizing that the compressed version of Y may not be available at the destination. This is denoted as $T_x : (bR_x|Y)$ in Table I. The details will be discussed in Section III.
- $Mode\ D$: This mode improves over $Mode\ C$ by enabling T_x to help T_y by cooperation so that reliability of T_y's link (and in return the effectiveness of T_x's compression) is improved.
- $Mode\ E$: This mode is the most general one which combines distributed compression and two sided cooperation to further improve the performance.

Finding a general rate-distortion region for distributed compression when the compressed streams may be lost is a difficult problem. That's why in $Mode\ C$ we concentrate on the asymmetric scenario where Y is compressed separately and X is compressed with respect to Y. Also for Gaussian sources, having side information at both the source encoder and the decoder does not improve the rate distortion performance over having it only at the decoder [6]. Hence in the above strategies, cooperation is not used to obtain side information at the encoder, it is mainly used to increase the channel reliability.

III. RATE DISTORTION WHEN LOSSY SIDE INFORMATION MAY BE ABSENT

Before computing the expected distortions corresponding to the five transmission modes, in this section we first review the prior work on rate-distortion function for Gaussian sources when lossless side information may be absent. Then we generalize to the case when the side information itself is compressed.

Without loss of generality, we can write, $Y = aX + Z$ when X and Y are jointly Gaussian with correlation matrix in (1). Here $Z \sim N(0, \sigma_z^2)$ is independent of X with $\sigma_z^2 = \sigma_y^2 - a^2\sigma_x^2$ and $a = \rho\frac{\sigma_y}{\sigma_x}$. Suppose Y is not available at X's encoder, and may or may not be available at the X's decoder. Let D_1 denote the squared error distortion achieved when Y is present at the destination, D_2 denote the distortion achieved when Y is absent. The minimum rate required to achieve the distortion pair (D_1, D_2) is [14],

$$R(D_1, D_2) = \begin{cases} \frac{1}{2}\ln\left(\frac{\sigma_x^2\sigma_z^2}{D_1(a^2D_2+\sigma_z^2)}\right) & \text{if} \quad D_1 \le \sigma_1^2, D_2 \le \sigma_x^2 \\ \frac{1}{2}\ln\left(\frac{\sigma_x^2}{D_2}\right) & \text{if} \quad D_1 > \sigma_1^2, D_2 \le \sigma_x^2 \\ \frac{1}{2}\ln\left(\frac{\sigma_x^2\sigma_z^2}{D_1(a^2\sigma_x^2+\sigma_z^2)}\right) & \text{if} \quad D_1 \le \sigma_1^2, D_2 > \sigma_x^2 \\ 0 & \text{if} \quad D_1 > \sigma_1^2, D_2 > \sigma_x^2 \end{cases} \quad (2)$$

where $\sigma_1^2 = \frac{D_2\sigma_z^2}{a^2D_2+\sigma_z^2}$

In the distributed compression setup with unreliable links in Figure 2, Y is also compressed. Using the optimum forward

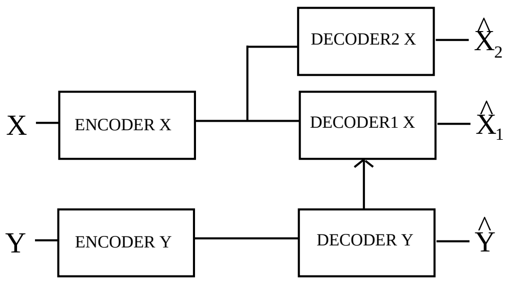

Fig. 2. Compression when lossy side information may be absent

channel model for Gaussian sources [15], we can express the description $\hat{Y}$ as;

$$\hat{Y} = \beta(Y+W) = \beta(aX+Z+W) \quad (3)$$

where $\beta = (1-\frac{D_y}{\sigma_y^2})$, and $W \sim N(0,\sigma_w^2)$ is independent of Y with $\sigma_w^2 = \frac{\sigma_y^2 D_y}{\sigma_y^2 - D_y}$. Here D_y refers to the distortion for Y. Note that if Y is compressed using $\overline{R}_y$ bits per source sample, then we have

$$D_y(\overline{R}_y) = \sigma_y^2 2^{-2\overline{R}_y} \quad (4)$$

Using the formulation in (2) and replacing Y with $\hat{Y}$, the rate distortion function of X can be expressed as

$$R(D_1,D_2) = \begin{cases} \frac{1}{2}\ln(\frac{\sigma_x^2(\sigma_z^2+\sigma_w^2)}{D_1(a^2D_2+\sigma_z^2+\sigma_w^2)}) & \text{if} \quad D_1 \le \sigma_2^2, D_2 \le \sigma_x^2 \\ \frac{1}{2}\ln(\frac{\sigma_x^2}{D_2}) & \text{if} \quad D_1 \ge \sigma_2^2, D_2 \le \sigma_x^2 \\ \frac{1}{2}\ln(\frac{\sigma_x^2(\sigma_z^2+\sigma_w^2)}{D_1(a^2\sigma_x^2+\sigma_z^2+\sigma_w^2)}) & \text{if} \quad D_1 \le \sigma_2^2, D_2 > \sigma_x^2 \\ 0 & \text{if} \quad D_1 \ge \sigma_2^2, D_2 > \sigma_x^2 \end{cases} \quad (5)$$

where $\sigma_2^2 = \frac{D_2(\sigma_z^2+\sigma_w^2)}{a^2D_2+\sigma_z^2+\sigma_w^2}$, $\sigma_w^2 = \frac{\sigma_y^2 D_y}{\sigma_y^2 - D_y}$.

We are mainly interested in the compression rate of X in the regime $D_1 \le \sigma_2^2$, $D_2 \le \sigma_x^2$ which can be expressed as

$$\overline{R}_x(D_1,D_2,D_y) = \frac{1}{2}\ln(\frac{\sigma_x^2(\sigma_z^2\sigma_y^2+a^2\sigma_x^2D_y)}{D_1(a^2D_2\sigma_y^2-a^2D_2D_y+\sigma_z^2\sigma_y^2+a^2\sigma_x^2D_y})$$

Alternatively the distortion D_1 for a given R_x, D_2 and D_y is equal to

$$D_1(\overline{R}_x,D_2,D_y) = \frac{\sigma_x^2(\sigma_z^2\sigma_y^2+a^2\sigma_x^2D_y)}{a^2D_2\sigma_y^2-a^2D_2D_y+\sigma_z^2\sigma_y^2+a^2\sigma_x^2D_y} 2^{-2\overline{R}_x}$$

where $\overline{R}_x$ is the compression rate of X in bits per sample.

IV. Expected end-to-end Distortion Calculation

For a given communication mode, average channel SNRs, source correlation and bandwidth ratio, the expected distortion is a function of the source rates, the amount of channel coding and level of cooperation. We will assume that a complete frame will be discarded if the channel decoder can not correct all the errors.

Assuming that we have channel codes operating at rates R_x, R_y bits per channel use with corresponding compression rates $\overline{R}_x$=bR_x and $\overline{R}_y$=bR_y bits per source sample and using our formulation in Sec III, the average distortions for the most general strategy, $Mode\ E$, can be expressed in terms of error/success probabilities as

$$\begin{aligned} ED_x = & P^1\ D_1(bR_x,D_2,D_y) \\ & + P^2\ D_2 \\ & + P^3\ D_1(0,D_2,D_y) \\ & + P^4\ \sigma_x^2 \end{aligned} \quad (6)$$

$$\begin{aligned} ED_y = & (P^1+P^3)D_y(bR_y) \\ & + (P^2+P^4)\sigma_y^2 \end{aligned} \quad (7)$$

In the above formulation, D_y is given by (4). Also D_2 is the target distortion for X if the description of Y is lost. Furthermore the probabilities P^i are defined as $P^i = \sum_{j=1}^{4} P^{i,j}$ with $P^{i,j}$ being the average probability of state (i,j) , where i and j denote whether (X,Y) is received and whether cooperation takes place, respectively. Here $i,j \in \{1,2,3,4\}$. For $i=1$, the compressed bits of both T_x and T_y are received, for $i=2$ the compressed bits of T_x are received but the bits for T_y are lost, $i=3$ means the compressed bits of T_y are received but the bits for T_x are lost and finally for $i=4$ the compressed bits of both T_x and T_y fail to reach the destination. The index j denotes whether cooperation successfully takes place or not, for instance, $j=1$ means both T_x and T_y are able to utilize cooperation. For $j=2$, only T_x receives help from T_y but not vice versa. Similarly, $j=3,4$ can be described. Note that probabilities $P^{i,j}$ depend on rates R_x, R_y and cooperation levels ϕ_x, ϕ_y.

In order to compute the average probabilities $P^{i,j}$, we consider an information theoretic approach. Considering complex Gaussian codebooks, for a channel code operating at a rate R bits per channel use, information is lost when the instantaneous channel capacity is lower than R, leading to the outage probability $P_{out} = Pr\{C(|h^2|SNR) < R\}$ for a point to point link where $C(x) = \log(1+x)$ is the Gaussian channel capacity and $|h|$ is the fading amplitude.

We will illustrate the computation of $P^{1,1}$ as an example. Using the outage approach for a given fading level h_{xy}, we have the inter-user channel from T_x to T_y is not in error if

$$R_x < \phi_x C(|h_{xy}^2|SNR_{xy})$$

Note that the effective channel-coding rate in the inter-user channel is $\frac{R_x}{\phi_x}$.

Similarly the information from T_y is received at T_x if

$$R_y < \phi_y C(|h_{xy}^2|SNR_{xy})$$

Given T_x and T_y cooperate, the compressed bits of both T_x and T_y are correctly received at the destination if

$$\begin{aligned} R_x < \phi_x C(|h_x^2|SNR_x) + (1-\phi_x)C(|h_y^2|SNR_y), \\ R_y < \phi_y C(|h_y^2|SNR_y) + (1-\phi_y)C(|h_x^2|SNR_x) \end{aligned}$$

Note that we have modeled the transmission of additional parity bits as independent Gaussian codebooks. Combining,

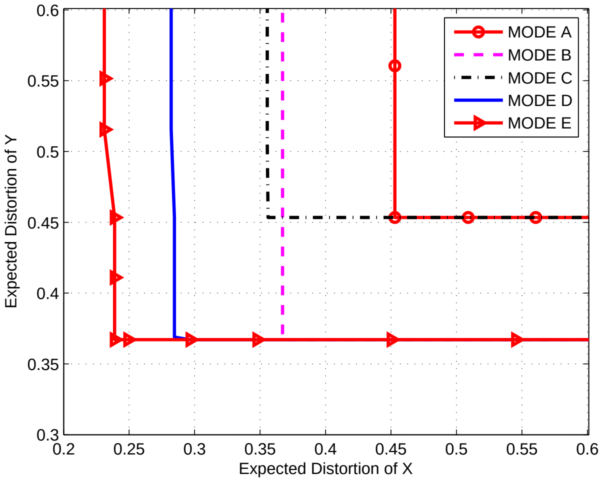

Fig. 3. ED_x versus ED_y , $\rho = 0.9$, SNR_x = SNR_y = 5dB, SNR_{xy} = 35dB

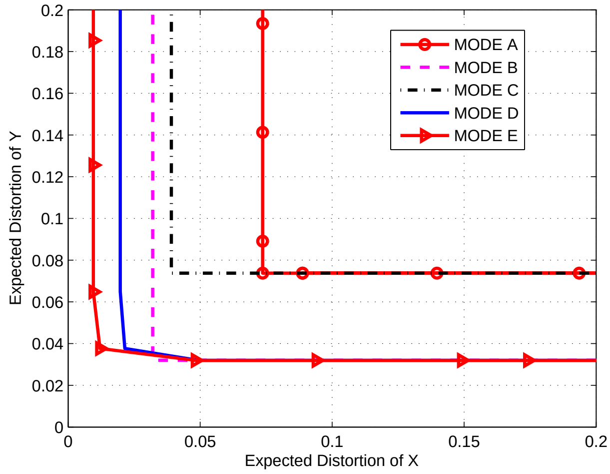

Fig. 4. ED_x versus ED_y , $\rho = 0.9$, SNR_x = SNR_y = 20dB, SNR_{xy} = 50dB

we have

$$\begin{aligned}P^{1,1} = Pr\{&R_x < \phi_x C(|h_x^2|SNR_x) + (1-\phi_x)C(|h_y^2|SNR_y),\\ &R_y < \phi_y C(|h_y^2|SNR_y) + (1-\phi_y)C(|h_x^2|SNR_x),\\ &R_y < \phi_y C(|h_{xy}^2|SNR_{xy}),\\ &R_x < \phi_x C(|h_{xy}^2|SNR_{xy})\}\end{aligned}$$

We can compute other probabilities $P^{i,j}$ similarly and minimize (6) and (7) over all choices of R_x, R_y, ϕ_x, ϕ_y and D_2. This expected distortion minimization problem will be numerically carried out in Section V.

We note that in order to obtain expected distortions for $Mode\ D$, we set $\phi_x = 1$. For $Mode\ C$, $\phi_x = \phi_y = 1$. For $Mode\ B$, we do not use the correlation at T_x, leading to

$$\begin{aligned}ED_x &= (P^1 + P^2)D_x(bR_x)\\ &+ (P^3 + P^4)\sigma_x^2 \qquad (8)\end{aligned}$$

For $Mode\ A$ we use the ED_x from (8) and set $\phi_x = \phi_y = 1$.

V. Results

In this section, we carry out the minimization of (6) and (7) numerically and compare the expected distortions achieved by different modes for various source correlation and channel link qualities. We assume $\sigma_x^2 = \sigma_y^2 = 1$ and $b = 1$. We first consider a symmetric scenario where the terminals are very close to each other such that $SNR_x = SNR_y = SNR$, $SNR_{xy} = SNR + 30dB$.

In order to observe the individual and joint effects of cooperation and distributed compression, for a fixed SNR and correlation coefficient, we vary R_x, R_y, ϕ_x, ϕ_y and D_2, compute the corresponding (ED_x,ED_y) pairs and plot the minimum values. Figure 3 and Figure 4 illustrate the ED_x versus ED_y behavior for a high correlation coefficient ($\rho = 0.9$), for two different channel signal to noise ratios, SNR=5dB and SNR=20dB, respectively. Comparison of $Mode\ A$ and $Mode\ B$ illustrates the effect of cooperation among terminals, and comparison of $Mode\ A$ and $Mode\ C$ shows how correlation helps to improve the distortion of X. Comparing $Mode\ D$ and $Mode\ C$, we argue that when only T_y makes use of cooperation, X also improves its distortion since the side information has higher chance of reaching the destination. This can be a further incentive for T_x to help T_y. For low SNR and high correlation, we observe that, utilizing the correlation only ($Mode\ C$) decreases the ED_x more than using cooperation only ($Mode\ B$). For SNR=5dB, $Mode\ E$ provides 49% reduction of ED_x and 19% reduction of ED_y compared to direct transmission ($Mode\ A$) and for SNR=20dB, we observe 87% reduction of ED_x and 56% reduction of ED_y .

Note that a chosen rate pair jointly affects both ED_x and ED_y when distributed compression is used (Modes C-E). One way to determine an optimal assignment of rates and the distortion trade off is to minimize the average distortion of X and Y, $\frac{ED_x+ED_y}{2}$. Figure 5 illustrates the achievable minimal average distortion with such optimal bit allocation over a wide range of channel SNRs for a high correlation coefficient ($\rho = 0.9$).

We also considered the case where inter-user channel is not as good as previously assumed. Specially, we assume $SNR_x = SNR_y = SNR$ and $SNR_{xy} = SNR$. Figure 6 shows the average distortion for different channel SNRs when $\rho = 0.9$. Comparing with Figure 5, we observe that now cooperation is less effective and distributed compression is more beneficial.

Figure 7 illustrates the minimum distortion of X as a function of correlation coefficient ρ. Comparing $Mode\ B$ and $Mode\ C$, we observe that for low source correlation, the main improvement is due to cooperation. For ρ >0.92, only having distributed compression results in better expected distortion than cooperation alone. Comparison of $Mode\ C$ and $Mode\ D$ further reinforces our observation that helping T_y improves the performance of T_x as well. Finally $Mode\ E$ outperforms all others over the entire range of ρ.

Our results are obtained for $b = 1$. For higher bandwidth

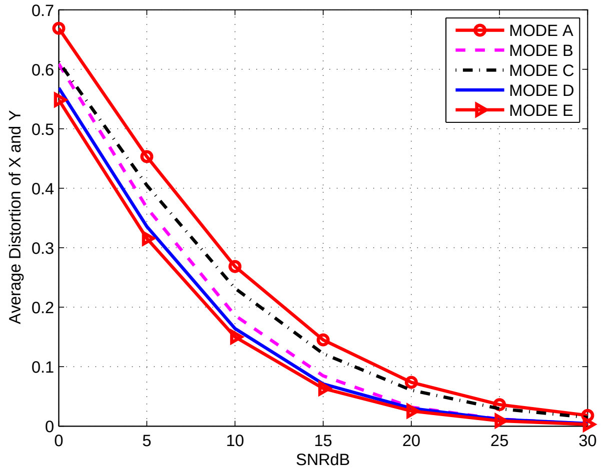

Fig. 5. Average distortion with $\rho = 0.9$, SNR_x = SNR_y = SNR, SNR_{xy} = SNR + 30dB

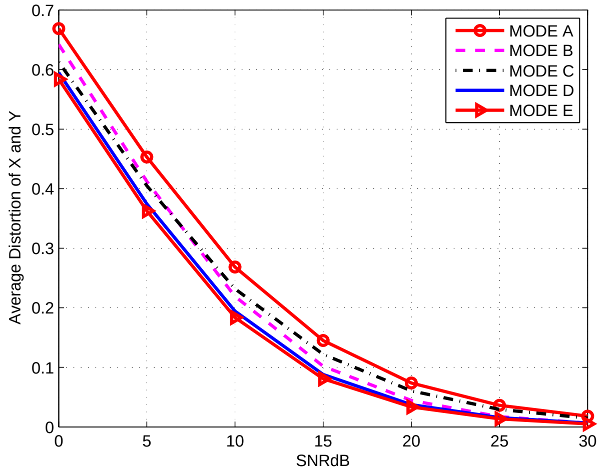

Fig. 6. Average distortion with $\rho = 0.9$, SNR_x = SNR_y = SNR_{xy} = SNR

ratios, the improvement due to cooperation will be larger [11]. However, we expect the impact of correlation to decrease as b increases.

VI. Conclusion

In this paper, we propose compression and transmission strategies which make use of source correlation as well as user cooperation. We argue that cooperative communication improves the performance of distributed compression by providing a more reliable side information at the destination. Overall, coupling multiterminal source coding with cooperative channel coding results in lower end-to-end source distortions for all the cooperating terminals.

This paper only considers a single source layer. Our prior work [11] illustrates the benefits of multiple source layers and unequal error protection through cooperation. A future direction is to extend the analysis here to the case when

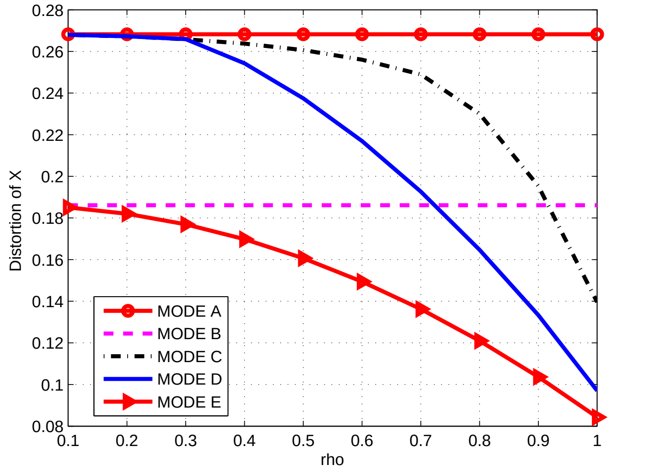

Fig. 7. D_x versus ρ with SNR_x = SNR_y = 10dB, SNR_{xy} = 40dB

each terminal uses layered compression. Another direction is to study symmetric distributed compression techniques that provide similar benefits to X and Y when link losses exist.

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